Exercise 11.1 (Revised) - Chapter 11 - Areas Related To Circles - Ncert Solutions class 10 - Maths

Updated On 11-02-2025 By Lithanya

NCERT Class 10 Maths Solutions: Chapter 11 - Areas Related To Circles

Unless stated otherwise, take $\pi = \frac{22}{7}$.

Ex 11.1 Question 1.

Find the area of a sector of a circle with radius $6\ \mathbf{cm}$, if angle of the sector is 60°.

Answer.

Here, $r = 6\ \text{cm}$ and $\theta = 60^\circ$

Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7} \text{cm}^2$

Ex 11.1 Question 2.

Find the area of a quadrant of a circle whose circumference is $22\ \text{cm}$.

Answer

Given, $2\pi r = 22\ \text{cm}$

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow r = \frac{7}{2} \text{cm}$

We know that for quadrant of circle, $\theta = 90^\circ$

$\therefore$ Area of quadrant $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8} \text{cm}^2$

Ex 11.1 Question 3.

The length of the minute hand of a clock is $14\ \text{cm}$. Find the area swept by the minute hand in 5 minutes.

Answer.

Here, $r = 14\ \text{cm}$ and since the minute hand rotates through $\frac{360^\circ}{60^\circ} = 6^\circ$ in one minute, therefore, angle swept by minute hand in 5 minutes $= \theta = 6^\circ \times 5 = 30^\circ$.

$$\therefore \text{ Area swept } = \frac{\theta}{360^\circ} \times \pi r^2$$

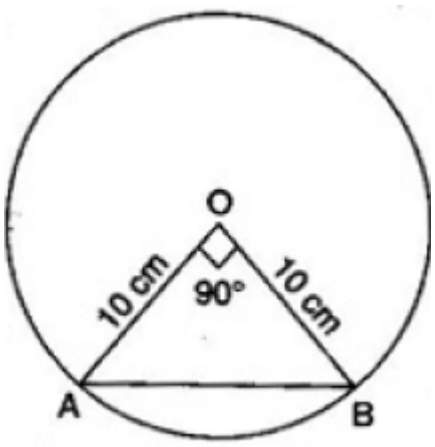
$$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 = \frac{154}{3} \text{cm}^2$$

Ex 11.1 Question 4.

A chord of a circle of radius $10\ \text{cm}$ subtends a right angle at the centre. Find the area of the corresponding: (i) minor segment, (ii) major segment. (Use $\pi = 3.14$)

Answer.

(i)Here, $r = 10\ \text{cm}$ and $\theta = 90^\circ$



$\therefore$ Area of minor sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 = 78.5 \text{ cm}^2$

Area of $\Delta OAB = \frac{1}{2} \times$ Base $\times$ Height

$= \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2$

$\therefore$ Area of minor segment = Area of minor sector - Area of $\triangle OAB$

$= 78.5 - 50 = 28.5 \text{ cm}^2$

(ii) For major sector, radius $= 10$ cm and $\theta = 360^\circ - 90^\circ = 270^\circ$

$\therefore$ Area of major sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{270^\circ}{360^\circ} \times 3.14 \times 10 \times 10 = 235.5 \text{ cm}^2$

Ex 11.1 Question 5.

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

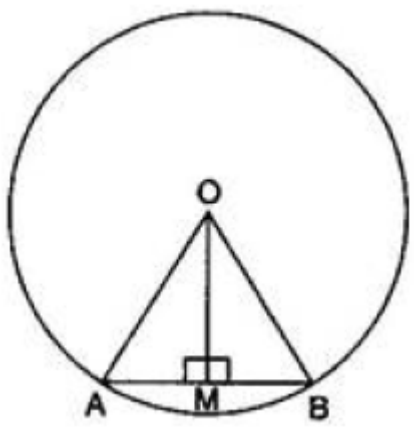
(i) the length of the arc.

(ii) area of the sector formed by the arc.

(iii) area of the segment formed by the corresponding chord.

Answer.

Given, $r = 21$ cm and $\theta = 60^\circ$



(i) Length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21 = 22 \text{ cm}$

(ii) Area of the sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 = 231 \text{ cm}^2$

(iii) Area of segment formed by corresponding chord

$= \frac{\theta}{360^\circ} \times \pi r^2 -$ Area of ΔOAB

$\Rightarrow$ Area of segment $= 231 -$ Area of $\triangle OAB$

In right angled triangle OMA and OMB,

$OM = OB$ [Radii of the same circle]

$OM = OM$[Common]

$\therefore \triangle OMA \cong \triangle OMB$[RHS congruency]

$\therefore AM = BM$[By C.P.C.T.]

$\therefore M$ is the mid-point of AB and $\angle AOM = \angle BOM$

$\Rightarrow \angle AOM = \angle BOM$

$= \frac{1}{2} \angle AOB = \frac{1}{2} \times 60^\circ = 30^\circ$

Therefore, in right angled triangle OMA,

$\cos 30^\circ = \frac{OM}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{21}$

$\Rightarrow \mathrm{OM}=\frac{21 \sqrt{3}}{2} \mathrm{cm}$

$\text{Also, } \sin 30^{\circ}=\frac{\mathrm{AM}}{\mathrm{OA}} \Rightarrow \frac{1}{2}=\frac{\mathrm{AM}}{21}$

$\Rightarrow \mathrm{AM}=\frac{21}{2} \mathrm{cm}$

$\therefore \mathrm{AB}=2 \mathrm{AM}=2 \times \frac{21}{2}=21 \mathrm{~cm}$

$\therefore \text{ Area of } \triangle \mathrm{OAB}=\frac{1}{2} \times AB \times OM$

$=\frac{1}{2} \times 21 \times \frac{21 \sqrt{3}}{2}=\frac{441 \sqrt{3}}{4} \mathrm{cm}^2$

Using eq. (i),

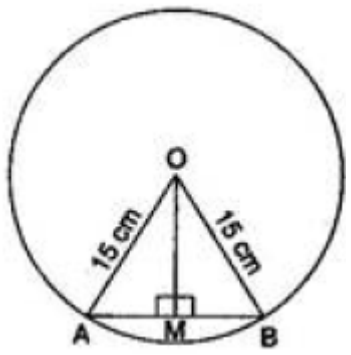
Area of segment formed by corresponding chord $=\left(231-\frac{441 \sqrt{3}}{4}\right) \mathrm{cm}^2$

Ex 11.1 Question 6.

A chord of a circle of radius $15 \mathrm{~cm}$ subtends an angle of 60° at the centre. Find the area of the corresponding minor and major segment of the circle.
$(\text{Use } \pi=3.14 \text{ and } \sqrt{3}=1.73)$

Answer.

Here, $r=15 \mathrm{~cm}$ and $\theta=60^{\circ}$



$\text{Area of minor sector } =\frac{\theta}{360^{\circ}} \times \pi r^2$

$=\frac{60^{\circ}}{360^{\circ}} \times 3.14 \times 15 \times 15$

$=117.75 \mathrm{~cm}^2$

For, Area of $\triangle AOB$,
Draw $\mathrm{OM} \perp \mathrm{AB}$.
In right triangles OMA and OMB,
$\mathrm{OA}=\mathrm{OB}$ [Radii of same circle]
$\mathrm{OM}=\mathrm{OM}[$ Common $]$
$\therefore \triangle \mathrm{OMA} \cong \triangle \mathrm{OMB}[\mathrm{RHS}$ congruency]
$\therefore \mathrm{AM}=\mathrm{BM}[$ By C.P.C.T. $]$
$\Rightarrow \mathrm{AM}=\mathrm{BM}=\frac{1}{2} \mathrm{AB}$ and
$\angle \mathrm{AOM}=\angle \mathrm{BOM}=\frac{1}{2} \angle \mathrm{AOB}=\frac{1}{2} \times 60^{\circ}=30^{\circ}$
In right angled triangle $\mathrm{OMA}, \cos 30^{\circ}=\frac{\mathrm{OM}}{\mathrm{OA}}$
$\Rightarrow \frac{\sqrt{3}}{2}=\frac{\mathrm{OM}}{15}$
$\Rightarrow \mathrm{OM}=\frac{15 \sqrt{3}}{2} \mathrm{cm}$
Also, $\sin 30^{\circ}=\frac{\mathrm{AM}}{\mathrm{OA}}$
$\Rightarrow \frac{1}{2}=\frac{\mathrm{AM}}{15} \Rightarrow \mathrm{AM}=\frac{15}{2} \mathrm{cm}$

$\Rightarrow 2 \mathrm{AM}=2 \times \frac{15}{2}=15 \mathrm{~cm}$

$\Rightarrow \mathrm{AB}=15 \mathrm{~cm}$

$\therefore \text{ Area of } \Delta \mathrm{AOB}=\frac{1}{2} \times AB \times OM$

$=\frac{1}{2} \times 15 \times \frac{15 \sqrt{3}}{2}=\frac{225 \sqrt{3}}{3}$

$=\frac{225 \times 1.73}{4}=97.3125 \mathrm{~cm}^2$

$\therefore \text{ Area of minor segment } = \text{ Area of minor sector } - \text{ Area of } \Delta \mathrm{AOB}$

$=117.75-97.3125=20.4375 \mathrm{~cm}^2$

$\text{And, Area of major segment } =\pi \mathrm{r}^2 - \text{ Area of minor segment}$

$=3.14 \times 15 \times 15-20.4375=706.5-20.4375=686.0625 \mathrm{~cm}^2$

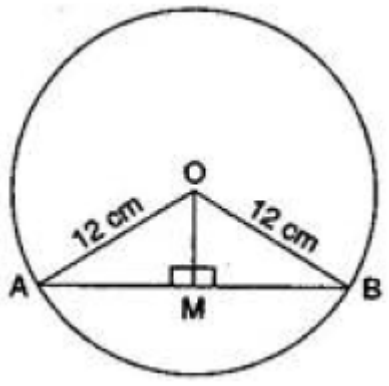
Ex 11.1 Question 7.

A chord of a circle of radius $12 \mathrm{~cm}$ subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.
(Use $\pi=3.14$ and $\sqrt{3}=1.73$)

Answer.

Here, $r = 12$ cm and $\theta = 120^\circ$

$\text{Area of corresponding sector } = \frac{\theta}{360^\circ} \times \pi r^2$



$= \frac{120^\circ}{360^\circ} \times 3.14 \times 12 \times 12$

$= 150.72 \text{ cm}^2$

For, Area of $\triangle \mathrm{AOB}$,

Draw $\mathrm{OM} \perp \mathrm{AB}$.

In right triangles OMA and OMB,

$\mathrm{OA} = \mathrm{OB}[$ [Radii of same circle]

$\mathrm{OM} = \mathrm{OM}$ [Common]

$\therefore \triangle \mathrm{OMA} \cong \triangle \mathrm{OMB}$ [RHS congruency]

$\therefore \mathrm{AM} = \mathrm{BM}$ [By C.P.C.T.]

$\Rightarrow \mathrm{AM} = \mathrm{BM} = \frac{1}{2}\mathrm{AB}$ and

$\angle \mathrm{AOM} = \angle \mathrm{BOM} = \frac{1}{2}\angle \mathrm{AOB} = \frac{1}{2} \times 120^\circ = 60^\circ$

In right angled triangle $\mathrm{OMA}, \cos 60^\circ = \frac{\mathrm{OM}}{\mathrm{OA}}$

$\Rightarrow \frac{1}{2} = \frac{OM}{12}$

$\Rightarrow OM = 6 \text{ cm}$

Also, $\sin 60^\circ = \frac{\mathrm{AM}}{\mathrm{OA}}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AM}{12}$

$\Rightarrow AM = 6\sqrt{3} \text{ cm}$

$\Rightarrow 2\mathrm{AM} = 2 \times 6\sqrt{3} = 12\sqrt{3} \text{ cm}$

$\Rightarrow \mathrm{AB} = 12\sqrt{3} \text{ cm}$

$\therefore \text{ Area of } \triangle \mathrm{AOB} = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3}$

$= 36 \times 1.73 = 62.28 \text{ cm}^2$

$\therefore \text{ Area of corresponding segment} = \text{Area of corresponding sector } - \text{ Area of } \triangle \mathrm{AOB}$

$= 150.72 - 62.28 = 88.44 \text{ cm}^2$

Ex 11.1 Question 8.

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find:

(i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 cm.

(Use $\pi = 3.14$)

Answer.

(i) Area of quadrant with 5 m rope

$= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 5 \times 5 = 19.625 \text{ m}^2$

(ii) Area of quadrant with 10 m rope

$= \dfrac{\theta}{360^\circ} \times \pi r^2$

$= \dfrac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 = 78.5 \text{ m}^2$

$\therefore$ The increase in grazing area

$= 78.5 - 19.625$

$= 58.875 \text{ m}^2$

Ex 11.1 Question 9.

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch.

Answer.

(i) Diameter of wire $= 35$ mm

$\Rightarrow$ Radius $= \frac{35}{2}$mm

$$\text{Circumference } = 2\pi r = 2 \times \frac{22}{7} \times \frac{35}{2}$$

$= 110 \text{ mm} \ldots\ldots\ldots \text{(i)}$

Length of 5 diameters $= 35 \times 5 = 175$ mm

$\therefore$ Total length of the silver wire required

$= 110 + 175 = 285$ mm

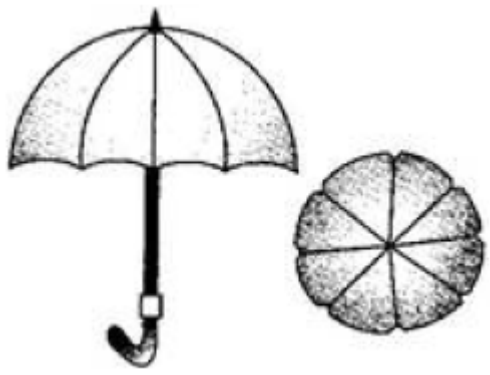
(ii) $r = \frac{35}{2}$mm and $\theta = \frac{360^\circ}{10} = 36^\circ$

$\therefore$ The area of each sector of the brooch $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{36^\circ}{360^\circ} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{385}{4}\text{mm}^2$

Ex 11.1 Question10.

An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Answer.

Here, $r = 45$ cm and

$\theta = \frac{360^\circ}{8} = 45^\circ$

Area between two consecutive ribs of the umbrella

$= \dfrac{\theta}{360^\circ} \times \pi r^2$

$= \dfrac{45^\circ}{360^\circ} \times \dfrac{22}{7} \times 45 \times 45$

$= \dfrac{22275}{28}\text{cm}^2$

Ex 11.1 Question 11.

A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Answer.

Here, $r = 25 \text{ cm}$ and $\theta = 115^\circ$

The total area cleaned at each sweep of the blades

$$= 2 \times \left(\frac{\theta}{360^\circ} \times \pi r^2 \right)$$

$$= 2 \times \left(\frac{115^\circ}{360^\circ} \times \frac{22}{7} \times 25 \times 25 \right)$$

$$= \frac{158125}{126} \text{cm}^2$$

Ex 11.1 Question 12.

To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of $16.5 \ \mathbf{km}$. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)

Answer.

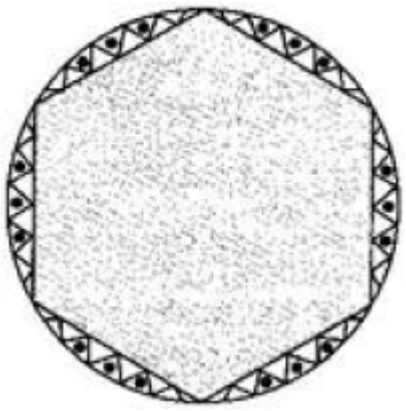
Here, $r = 16.5 \text{ km}$ and $\theta = 80^\circ$

$$\text{The area of sea over which the ships are warned } = \frac{\theta}{360^\circ} \times \pi r^2$$

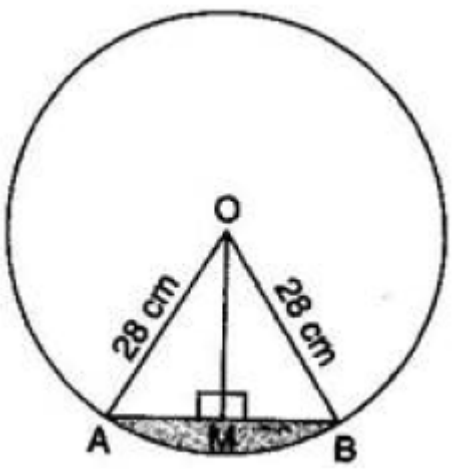
$$= \frac{80^\circ}{360^\circ} \times 3.14 \times 16.5 \times 16.5 = 189.97 \text{ km}^2$$

Ex 11.1 Question 13.

A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. $(\text{Use } \sqrt{3} = 1.7)$



Ans. $r = 28 \text{ cm}$ and $\theta = \frac{360^\circ}{6} = 60^\circ$



$$\text{Area of minor sector } = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28$$

$$= \frac{1232}{3} = 410.67 \text{ cm}^2$$

For, Area of $\triangle \text{AOB}$, Draw $\text{OM} \perp \text{AB}$.

In right triangles OMA and OMB,

$\text{OA} = \text{OB}$ [Radii of same circle]

$\text{OM} = \text{OM}$[Common]

$\therefore \triangle \text{OMA} \cong \triangle \text{OMB}$ [RHS congruency]

$\therefore \text{AM} = \text{BM}$[By C.P.C.T.]

$\Rightarrow \text{AM} = \text{BM} = \frac{1}{2}\text{AB}$ and $\angle \text{AOM} = \angle \text{BOM} = \frac{1}{2}\angle \text{AOB} = \frac{1}{2} \times 60^\circ = 30^\circ$

In right angled triangle OMA,

$$\cos 30^\circ = \frac{\text{OM}}{\text{OA}}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\text{OM}}{28}$$

$$\Rightarrow \text{OM} = 14\sqrt{3} \text{ cm}$$

Also, $\sin 30^\circ = \frac{\mathrm{AM}}{\mathrm{OA}}$

$\Rightarrow \frac{1}{2} = \frac{\mathrm{AM}}{28}$

$\Rightarrow \mathrm{AM} = 14 \text{ cm}$

$\Rightarrow 2\mathrm{AM} = 2 \times 14 = 28 \text{ cm}$

$\Rightarrow \mathrm{AB} = 28 \text{ cm}$

$\therefore$ Area of $\triangle \mathrm{AOB} = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 28 \times 14\sqrt{3} = 196\sqrt{3}$

$= 196 \times 1.7 = 333.2 \text{ cm}^2$

$\therefore$ Area of minor segment = Area of minor sector - Area of $\triangle AOB$

$= 410.67 - 333.2 = 77.47 \text{ cm}^2$

$\therefore$ Area of one design $= 77.47 \text{ cm}^2$

$\therefore$ Area of six designs $= 77.47 \times 6 = 464.82 \text{ cm}^2$

Cost of making designs $= 464.82 \times 0.35 =$ Rs. 162.68

Ex 11.1 Question 14.

Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius $\mathbf{R}$ is:

(A) $\frac{p}{180^\circ} \times 2\pi \mathrm{R}$

(B) $\frac{p}{180^\circ} \times \pi \mathrm{R}^2$

(C) $\frac{p}{360^\circ} \times 2\pi \mathrm{R}$

(D) $\frac{p}{720^\circ} \times 2\pi \mathrm{R}^2$

Answer.

(D) Given, $r = \mathrm{R}$ and $\theta = p$

$\text{Area of sector } = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{p}{360^\circ} \times \pi \mathrm{R}^2$

$= \frac{p}{2 \times 360^\circ} \times 2\pi \mathrm{R}^2$

$= \frac{p}{720^\circ} \times 2\pi \mathrm{R}^2$